

## Chapter 3 Advanced Enrichment

This packet belongs to \_\_\_\_\_

## Exponential growth and decay

Exponential growth happens when a quantity increases by a fixed rate each time period.

Exponential decay happens when a quantity decreases by a fixed rate each time period.

Exponential growth  $y = y_0(1 + r)^t$

Exponential decay  $y = y_0(1 - r)^t$

$y$  \_\_\_\_\_  $y_0$  \_\_\_\_\_  $r$  \_\_\_\_\_  $t$  \_\_\_\_\_

### Example 1

The country of Latvia has been experiencing a 0.6% annual decrease in population. In 2003, its population was 2,350,000. What would be Latvia's population in 2013 if the rate of decrease remained the same?

2. A municipal bond pays 5% per year. If \$2000 is invested in these bonds, what is the value of the investment after 4 yrs? 8yrs?

3. A farmer buys a tractor for \$60,000. If the tractor depreciates 10% per year, what is the value of the tractor after 8 years? 15 years? When will the tractor be worth \$0.

4. Computer use around the world has risen 19% annually since 1980. If 18.9 million computers were in use in 1980, predict the number of computers that will be in use in 2017.

1. The population of Mexico has been increasing at an annual rate of 1.7%. If the population of Mexico was 104,900,000 in the year 2003, predict the population of Mexico in 2019.
2. A car sells for \$24,000. If the annual rate of depreciation is 13%, what is the value of the car after 8 yrs? 50yrs? Is the 50 yr estimated value accurate?
3. A cup of coffee contains 130 mg of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how much caffeine will remain in the body after 3 hours?
4. Determine the amount of an investment if \$3000 is invested at an interest rate of 5.5% each year for 3 yrs.
5. The Smith family bought a condo for \$115,000. Assuming that the value of the condo will appreciate 5% each year, how much will the condo be worth in 6 years?
6. A piece of crystal weighing 3.7 grams depreciates at a steady rate of 98.5% per year. How much of the crystal is left after 15 years? Express the solution in scientific notation to the nearest tenth in kg.

## Predicting Populations with Exponential and Logistic Growth Functions

The natural exponential function is  $y = e^x$ . We will use this to calculate much more complex problems, problems that cannot be solved with the basic exponential growth/decay function.

To familiarize yourselves with the keystrokes on the calculator, evaluate the following...

a)  $e^3$

b)  $2e^{-0.53}$

c)  $2.55e^{.43}$

### Example 1

A radioactive substance decays in such a way that the amount of mass remaining after  $t$  days is modeled by the equation

$$y = 13e^{-0.015t}$$

where  $t$  is measured in kilograms.

- a) Find the mass at  $t = 0$
- b) How much of the mass remains after 45 days? 102 days?

### Example 2

An infectious disease begins to spread in a small city with a population of 10,000. After  $t$  days, the number of persons who have succumbed to the virus is modeled by the equation

$$y = \frac{10,000}{5 + 1245e^{-0.97t}}$$

- a) How many people are there initially (at  $t = 0$ )?
- b) Find the number of people infected after 1 day, 2 days, 31 days?

### Example 3

A sky diver jumps from a reasonable height above the ground. The air resistance she experiences is proportional to her velocity. The following equation can be used to model the downward velocity of the sky diver at time  $t$ .

$$y = 80(1 - e^{-0.2t})$$

where  $t$  is measured in seconds and  $y$  is measured in feet per second.

- a) Find the initial velocity of the sky diver.
- b) Find the velocity after 5 seconds and after 10 seconds.
- c) The maximum velocity of a falling object with wind resistance is called its terminal velocity. Can you use the equation to determine the terminal velocity?

### Example 4

Assume that a population of rabbits behaves according to the following model

$$y = \frac{300}{0.05 + \left(\frac{300}{n_o} - 0.05\right)e^{-0.55t}}$$

Where  $n_o$  is the initial rabbit population.

- a) If the initial population is 50 rabbits, what will be the population after 10 years?
- b) If the initial population is 75 rabbits, what will be the population after 100 years?
- c) If the initial population is 900 rabbits, what will be the population after 100 years?
- d) What does this tell you about the maximum number of rabbits that can live on the island?

- 1) When a certain medication is administered to a patient, the number of milligrams remaining in the patient's blood stream after  $t$  hours is modeled by

$$y = 50e^{-0.2t}$$

- a) How many milligrams of the medication remain in the patient's bloodstream after 3 hours?

- 2) Radioactive iodine is used as a tracer in diagnosing certain thyroid gland disorders. This type of iodine decays in such a way that the mass remaining after  $t$  days is modeled by

$$y = 6e^{-0.087t} \quad \text{where } y \text{ is measured in grams}$$

- a) Find the initial mass at time = 0  
b) Find the amount of iodine in milligrams that remains in your bloodstream after 45 days?

- 3) A 50-gallon tank is filled completely with pure water. Salt water with a concentration of 0.3 lb/gal is then pumped into the tank and the resulting mixture overflows at the same rate. The amount of salt in the tank is modeled by

$$y = 15(1 - e^{-0.04t})$$

where  $t$  is measured in minutes and  $y$  is measured in pounds.

- a) How much salt is in the tank after 5 min? 10 min?

- 4) The population of a certain species of birds is limited by the type of habitat required for nesting. The population is modeled by

$$y = \frac{5600}{0.5 + 27.5e^{-0.044t}}$$

where  $t$  is measured in years.

- a) Find the initial bird population.  
b) What is the population of birds after 5 years? 50 years? 500 years?  
c) What is the maximum number of birds that the island can inhabit?

- 5) Animal populations are not capable of unrestricted growth because of limited habitat and food supplies. Under such conditions, the population is modeled by

$$y = \frac{d}{1 + ke^{-ct}}$$

where  $c$ ,  $d$  &  $k$  are positive constants. For a certain fish population in a small pond  $d = 1200$ ,  $k = 11$ ,  $c = 0.2$  and  $t$  is measured in years. The fish were introduced into the pond at time = 0.

- a) How many fish were originally in the pond?  
b) What is the population of fish after 10 years? 30 years?

## Creating your own exponential models

Writing exponential models and making predictions is used in many fields from police work to banking and medical sciences. We will focus on writing a model in the form  $y = y_0 b^t$

Where  $y$  is the future amount,  $y_0$  is the initial amount,  $b$  is the growth rate and  $t$  = time.

### Example 1

The number of dandelions in your lawn increases by 5% a week and there are 75 dandelions now.

- a) Find the equation that models the data.
- b) How many dandelions will there be in 6 weeks?

### Example 2

The fruit fly population in a certain laboratory triples every day. Today there are 200 fruit flies.

- a) Find the equation that models the data.
- b) How many fruit flies will there be in 1 week?

### Example 3

At the beginning of an experiment a culture of bacteria contains 1000 bacteria. Five hours later, there are 7600 bacteria. Assume that the bacteria grow exponentially.

- a) Find the equation that models the data.
- b) How many bacteria are there after 24 hours?

#### Example 4

A weekly study of the frog population at Frog Tree Hollow Park gained the following results. At week 1, there were 18 frogs. At week 3, there were 162 frogs.

- a) Find the equation that models the data.
- b) How many frogs are there after 6 weeks? 45 days?

#### Example 5

A daily study of the termite population at the Herman household was made and the following data was collected. At day 1, there were 175 termites. At day 4, there were 2,734,375.

- a) Find the equation that models the data.
- b) How many termites were there after 3 days? 1 week?



## Half-life models

The half-life of a radioactive substance is the time it takes a given quantity to decay to one-half its original mass. The half-life depends only on the substance, not on the size of the sample. Since radioactive substances decay exponentially, their decay can be described using the same equation we used on the previous page with a slight adjustment.  $y = y_o * .5^{\left(\frac{t}{h}\right)}$  Where  $y$  is the future amount,  $y_o$  is the initial amount,  $h$  is the half-life of the substance and  $t$  is the time.

### Example 6

Suppose 100 lbs of plutonium is deposited at a nuclear waste site. The half-life of plutonium is 24,000 years. How much plutonium will be left after 100,000 years?

### Example 7

After a person is bit by a zombie, the virus Solanum spreads throughout their system. The half-life of Solanum is estimated at 71 days. The mass of the virus is varied but recent samples collected suggest an average mass of 57.34 milligrams. Studies done in laboratories within a secret location show that the person will be 7 times less likely to attack you if the virus has a mass of less than 5.47 milligrams. Should a zombie captured after 9 months have enough Solanum in their system to still attack you? Assume a 30 day month.

**Do all problems on a separate sheet of paper and show all work.**

1. At the beginning of an experiment, there were 200 h-pylori bacteria. An hour later, there were 205 bacteria. Assuming that the bacteria grow exponentially, how many will there be after 10 hours? 2 days?
2. A study of ants in a giant ant hill determined that the population was 67.4 million ants in 2010. The number of ants has been increasing exponentially by 2.6% each year. How many ants will there be in 2030?
3. There are currently 3.2 million people who own some form of video game system in the US. The number of people who own some form of video game system has been growing exponentially by 3.68% for many years now. How many people will own a video game system in 15 years?
4. A daily study of the cockroach population at the local diner was made and the following data was collected. At day 2, there were 98 cockroaches. At day 4, there were 4,802 cockroaches. If no one calls an exterminator, how many cockroaches will there be after 1 week?
5. The unicorn population above the United States on August 14<sup>th</sup> was 4,096. Mr. Herman and his trusty sidekick Andy went hunting on a weekly basis to rid the world of these horrible creatures. After 3 weeks, there were only 512 unicorns left. How many unicorns were there at the end of the 8<sup>th</sup> week? How many weeks will it take before there are no unicorns left?
6. The half-life of radium is 1620 years. Find the amount of 100mg of radium that will be left after 800yrs? 3200yrs?
7. Caffeine is found in many popular soft drinks and coffees. The half-life of caffeine is 5.7 hours. At 7 a.m., you consume one 16 ounce can of Monster. There is 160 mg. of caffeine in this serving. Find the amount of caffeine left in your bloodstream by 10:30 a.m.

**Do all problems on a separate sheet of paper and show all work.**

1. A population of bacteria grows by 35% every hour. If the population begins with 100 specimens, how many will there be after 6 hours? 3 days?
2. A study of bees in a giant beehive determined that the population was 32.4 million bees in 2011. The number of bees has been increasing exponentially by 3.5% each year. How many bees will there be in 2029?
3. A fully inflated beach ball loses 6% of its air every day. If the beach ball originally contained 4000 cubic centimeters of air, how many cubic centimeters of air will it hold after 10 days?
4. A yearly study of the alligator population in the everglades in Florida was made and the following data was collected. At year 2, there were 1,677 gators. By year 7, there were 1,802 gators. What is the growth rate? If this rate of growth remains constant, how many gators should there be in year 13?
5. The population of Chile in 1970 was 9.37 million. By the year 2000, the population was 15.16 million. Based on this rate of growth, what should be Chile's population in the year 2021?
6. The half-life of americium-241 is 432.2 years. Find the amount of 975mg of americium that will be left after 300yrs? 2,400yrs?
7. Benadryl is drug used to prevent allergy symptoms like itching, running nose, rashes on skin and cold. The half-life of Benadryl is 7.7 hours. At 7 a.m., you consume 15 ml of Benadryl to fight the allergy symptoms you are experiencing. Find the amount of Benadryl left in your bloodstream by 9:45 p.m.

### Advanced Bonus Opportunities for Chapter 3

- 1) Find the numeric palindrome greater than 12 and less than 32.
- 2) What is the additive inverse of  $-2$ ?
- 3) Simplify  $(y - 7) - |-y| + (-2 + y)$  for  $y = 6$
- 4) Simplify  $|-a - (a - b(a - b(a - b)))|$  for  $a = -3$  &  $b = -2$
- 5) When the digits of a two digit number are reversed, it is 9 less than 3 times the original number. List any possibilities as solutions to this question. Explain why there are or why there are not many different solutions.

- 6) What number is  $x$  in this magic square?

32	4	6	$y$
10	22	20	16
$z$	14	12	24
8	$x$	30	2

*Attempt as many problems as you can. Show all steps on all problems. Skipping steps will result in the loss of points. Do all work on a separate sheet of paper.*

- 7) What are the next 3 terms of this sequence? 5, 9, 13, 17, 21, ... Write an expression that can be used to find the 28<sup>th</sup> term. Use the expression to find the 28<sup>th</sup> term.
- 8) A bag contains 35 green, some black and three red marbles. If you reached blindfolded, the probability of picking red is  $\frac{1}{23}$ . How many black marbles are there?
- 9)  $\frac{.13 \times 10^5}{5 \times 10^2} = ?$
- 10)  $9(1.\bar{2}) = ?$
- 11) Evaluate  $b^{b+1} - x^y - ab + 5^a$  for  $x=1, y=5, a=0$  &  $b=2$
- 12) The faucet of a certain tub fills in 15 minutes. The shower head fills the tub in 20 minutes. The drain empties the tub in 12 minutes. If all the faucets are turned on and the drain is left open, how long—if at all—will it take to fill the tub?
- 13)  $\frac{7(10^{10^{100}})}{\text{googolplex}(25)} = ?$
- 14) Find the difference  $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$