Chapter 3 Advanced Common Core

Exponents

This packet belongs to _____

Adding & Subtracting Integers

When adding two integers, there are four possible problems you can encounter.

1) Both numbers are positive. 3 + 3 or 2 + 56

2) Both are negative. -2 + -4 or -5 +-12

3) One is positive and the other is negative -8 + 4 or 3 + -5

4) One of the two numbers is zero. 0 + 2 or -7 + 0

Each of these are solved differently. What are some basic rules that apply to these problems?

Examples

$-3+8$ $2+-4$ $-6+-7$ $-1+2+-4+5+-7$

When subtracting integers, change the subtraction sign to an addition sign and change the sign of the number that follows.

Examples

$7-(-3)$ $-4-(-3)$ $0-(+5)$ $-9-(-4)-3-(-7)$

Multiplying & Dividing Integers

The rules are the same for multiplying as they are for dividing integers.

Try these examples to find the two rules...

Examples

$2 \cdot 4x$ $\quad -3 \cdot -7$ $\quad \frac{5}{2}$ $\quad \frac{-8h}{-4}$

Rule 1:

Examples

$2 \cdot -4$ $\quad 3 \cdot -7a$ $\quad \frac{-5}{2c}$ $\quad \frac{8}{-4}$

Rule 2:

When multiplying or dividing several terms, an even number of negative signs will yield a _____ product or quotient, an odd number of negative signs will give you a _____ product or quotient.

Example 1

$$\frac{3(-2) \cdot 2x(-4)(-2)}{-5(-3)(7)(d)}$$

Example 2

$$\frac{3(-7) \cdot 2x(-3x)(-4y)}{(-6x)(21y)(x)}$$

Example 3.

$$\frac{-3x(2x)+(-3x)}{(-3x)(-2x)}$$

2.

Negative Exponents

2^4	
2^3	
2^2	
2^1	
2^0	
2^{-1}	
2^{-2}	
2^{-3}	
2^{-4}	

Words	Numbers	Algebra
A power with a negative exponent equals _____ by that power with its _____ exponent.		

1. 3^{-2} 2. -3^{-2} 3. $(-3)^{-2}$

4. 1^{-5} 5. 5^{-1} 6. -5^{-1}

7. $(-2)^{-3}(3)^{-2}$ 8. $2^{-3}(2)^{2}$

9. $\frac{1}{5^{-2}}$ 10. $\frac{2^{3}}{2^{-3}}$ 11. $\frac{3^{-2}}{3^{2}}$

Negative Exponents

For every number a ≠ 0 $a^{-m} =$

Examples

5^{-3} 2^{-1} x^{-3} $\left[\frac{1}{3}\right]^{-3}$ $(x-2)^{-4}$ $(-.42)^{-2}$

Express in simplest form. Use >,< or = to compare. Show all work on a separate piece of paper.

1.	4^3	>	5^2	2.	2^{-3}	_____	6^{-1}
3.	3^2	_____	4^0	4.	$(-8)^{-4}$	_____	6^{-3}
5.	$(-2)^{-2}$	_____	2^2	6.	9^{-2}	_____	3^{-4}
7.	12^{-3}	_____	$(-11)^{-3}$	8.	$(-11)^0$	_____	$(5)^{-5}$
9.	10^{-4}	_____	$(-10)^3$	10.	6^{-2}	_____	$(-12)^{-2}$
11.	8^0	_____	$(-9)^0$	12.	9^{-3}	_____	11^{-2}
13.	$(-4)^{-4}$	_____	$(-4)^0$	14.	2^{-5}	_____	$(-0.3)^2$
15.	$(-0.7)^2$	_____	$(0.2)^4$	16.	$\left[\frac{3}{4}\right]^{-2}$	_____	$(-10)^{-1}$
17.	$\left[\frac{-8}{9}\right]^{-2}$	_____	0.3^3	18.	$(-3)^4$	_____	$\left[\frac{1}{2}\right]^{-2}$
19.	$\left[\frac{7}{12}\right]^{-3}$	_____	4^2	20.	$\left[\frac{4}{5}\right]^{3}$	_____	0.7^4
21.	$\left[\frac{3}{4}\right]^{-3}$	_____	$(-0.06)^3$	22.	$(-0.8)^{-3}$	_____	$\left[\frac{-8}{9}\right]^{2}$
23.	Evaluate $(4^{-2})(4^2)$			24.	Evaluate $\frac{(a^{-5}b^4)^3}{a^{-5}b^2}$		

Exponential Expressions

Adding and subtracting examples

$2(a^3)+5(a^3)$ $3(5^2)+2(5^2)$

$7(2^5)-6(2^5)$ $5(2^4)-5(2^3)$

Multiplying examples

a^3a^5 4^54^{-3} $3c^24c^5$ $-6y(3y^3)$

Dividing examples

$\frac{3^3}{3^2}$ $\frac{x^{-4}}{x^4}$ $\frac{6z^4}{3z^2}$ $\frac{4a^3}{16a^{-2}}$

Raising exponents to a power

$(y^3)^2$ $(3^4)^5$ $(2^2)^{-3}$ $(x^{-3})^{-2}$

$$\frac{(9x^{-2}y^3)^2}{(6xy^2)^3}$$

$$(4x^{-1}y^{-2})^2(2x^4y^5)^3$$

Simplify. Show all work on a separate sheet of paper.

1. $5(-2)^2$
2. $3(-1)^3$
3. $-\frac{1}{2}(-4)^2$
4. $3(3)^2+5(3)^2$
5. $3x^2-5x^2$
6. $6c^0-3c^0$
7. x^3x^{-2}
8. $-7a^2(5x^3)$
9. $2^{-1}\cdot 2^3\cdot 2^{-5}\cdot 2^2$
10. $\dfrac{2^4}{2^{-2}}$
11. $\dfrac{2x^5}{x^{-2}}$
12. $\dfrac{2a^{-2}}{4a^2}$
13. $\dfrac{3x^4}{2x^{-7}}$
14. $(5^2)^5$
15. $(y^3)^{-1}$
16. $(a^{-2})^{-3}$
17. $(c^4)^0$
18. $(x^2)^3$
19. $\dfrac{(3x-2)^4}{(3x-2)^{-3}}$
20. $(2x^3+4)^{-6}(2x^3+4)^4$
21. $\left(\dfrac{(x+3)^{-2}}{y^{-4}}\right)^{-3}$

Write an exponential equation to solve

22. A number squared plus six squared equals one hundred. What is the number?

23. Four times a number squared minus that number squared is seventy-five. What is the number?

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8-1

Multiplying Monomials

Multiply Monomials A **monomial** is a number, a variable, or a product of a number and one or more variables. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 **Simplify $(3x^6)(5x^2)$.**

Example 2 **Simplify $(-4a^3b)(3a^2b^5)$.**

Exercises

Simplify.

1. $y(y^5)$
2. $n^2 \cdot n^7$
3. $(-7x^2)(x^4)$
4. $x(x^2)(x^4)$
5. $m \cdot m^5$
6. $(-x^3)(-x^4)$
7. $(2a^2)(8a)$
8. $(rs)(rs^3)(s^2)$
9. $(x^2y)(4xy^3)$
10. $\frac{1}{3}(2a^3b)(6b^3)$
11. $(-4x^3)(-5x^7)$
12. $(-3j^2k^4)(2jk^6)$
13. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$
14. $(-5xy)(4x^2)(y^4)$
15. $(10x^3yz^2)(-2xy^5z)$

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Multiplying Monomials

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. $\frac{21a^2}{7b}$

2. $\frac{b^3c^2}{2}$

Simplify.

3. $(-5x^2y)(3x^4)$

4. $(2ab^2c^2)(4a^3b^2c^2)$

5. $(3cd^4)(-2c^2)$

6. $(4g^3h)(-2g^5)$

7. $(-15xy^4)\left(-\frac{1}{3}xy^3\right)$

8. $(-xy)^3(xz)$

9. $(-18m^2n)^2\left(-\frac{1}{6}mn^2\right)$

10. $(0.2a^2b^3)^2$

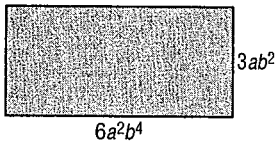
11. $\left(\frac{2}{3}p\right)^2$

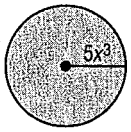
12. $\left(\frac{1}{4}cd^3\right)^2$

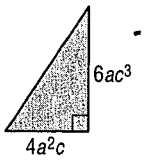
13. $(0.4k^3)^3$

14. $[(4^2)^2]^2$

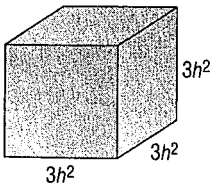
GEOMETRY Express the area of each figure as a monomial.

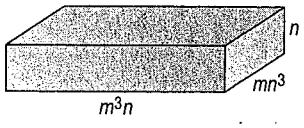
15. 

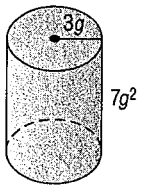
16. 

17. 

GEOMETRY Express the volume of each solid as a monomial.

18. 

19. 

20. 

21. **COUNTING** A panel of four light switches can be set in 2^4 ways. A panel of five light switches can set in twice this many ways. In how many ways can five light switches be set?

22. **HOBBIES** Tawa wants to increase her rock collection by a power of three this year and then increase it again by a power of two next year. If she has 2 rocks now, how many rocks will she have after the second year?

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8-2

Dividing Monomials

Quotients of Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 **Simplify $\frac{a^4b^7}{ab^2}$. Assume neither a nor b is equal to zero.**

Example 2 **Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that b is not equal to zero.**

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example **Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that the denominator is not equal to zero.**

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8-2

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{8^8}{8^4}$

2. $\frac{a^4b^6}{ab^3}$

3. $\frac{xy^2}{xy}$

4. $\frac{m^5np}{m^4p}$

5. $\frac{5c^2d^3}{-4c^2d}$

6. $\frac{8y^7z^6}{4y^6z^5}$

7. $\left(\frac{4f^3g}{3h^6}\right)^3$

8. $\left(\frac{6w^5}{7p^6s^3}\right)^2$

9. $\frac{-4c^2}{24c^5}$

10. $x^3(y^{-5})(x^{-8})$

11. $p(q^{-2})(r^{-3})$

12. 12^{-2}

13. $\left(\frac{3}{7}\right)^{-2}$

14. $\left(\frac{4}{3}\right)^{-4}$

15. $\frac{22r^3s^2}{11r^2s^{-3}}$

16. $\frac{-15w^0u^{-1}}{5u^3}$

17. $\frac{8c^3d^2f^4}{4c^{-1}d^2f^{-3}}$

18. $\left(\frac{x^{-3}y^5}{4^{-3}}\right)^0$

19. $\frac{6f^{-2}g^3h^5}{54f^{-2}g^{-5}h^3}$

20. $\frac{-12t^{-1}u^5v^{-4}}{2t^{-3}uv^5}$

21. $\frac{r^4}{(3r)^3}$

22. $\frac{m^{-2}n^{-5}}{(m^4n^3)^{-1}}$

23. $\frac{(j^{-1}k^3)^{-4}}{j^3k^3}$

24. $\frac{(2a^{-2}b)^{-3}}{5a^2b^4}$

25. $\left(\frac{q^{-1}r^3}{qr^{-2}}\right)^{-5}$

26. $\left(\frac{7c^{-3}d^3}{c^5de^{-4}}\right)^{-1}$

27. $\left(\frac{2x^3y^2z}{3x^4yz^{-2}}\right)^{-2}$

28. **BIOLOGY** A lab technician draws a sample of blood. A cubic millimeter of the blood contains 22^3 white blood cells and 22^5 red blood cells. What is the ratio of white blood cells to red blood cells?

29. **COUNTING** The number of three-letter "words" that can be formed with the English alphabet is 26^3. The number of five-letter "words" that can be formed is 26^5. How many times more five-letter "words" can be formed than three-letter "words"?

The Fundamental Counting Principle

the number of ways one activity can be performed	X	the number of ways a second activity can be performed	=	the total number of ways both activities can be performed together

Examples

1. Kelly has 6 shirts and 4 coordinating pairs of pants. What is the possible number of shirt-pants outfits?

2. The menu for dinner lists 2 soups, 4 meats and 3 desserts. How many different meals that have one soup, one meat and one dessert are possible?

3. How many different 5-lettter "words" are possible using the letters of TRIANGLE? Each letter can only be used once. If a "word" is selected at random, what is the probability that it is the word ANGLE?

4. What is the probability that the 5 letter "word" in the previous problem does not contain the letter G?

5. Employee identification codes at a company contain 2 letters followed by 2 numbers. How many possible codes are there? What is the probability of being assigned the code MT49? The company decides that it wants to omit the letters I and O because they look too much like 1 and zero. Based on this new change, what is the probability that you get assigned the code MT49? BX34? DO21?

6. Based on the change in the previous problem, what is the probability that your code does not contain a 2 or a 5?

1) Shoppers in a large shopping mall are categorized as male or female, over 30 or 30 and under, and cash or credit card shoppers. In how many ways can the shoppers be categorized?

2) A car model comes in nine colors, with or without air conditioning, with or without a sunroof, with or without an automatic transmission, and with or without antilock brakes. In how many ways can the car be ordered?

3) How many different four-letter radio station call letters can be formed if the first letter must be W or K?

4) A stock can go up, go down, or stay unchanged. How many possibilities are there if you own seven stocks?

5) A social security number contains nine digits, such as 000-00-0000. How many different social security numbers can be formed?

6) You are about to take a seven-question multiple choice test. Each of these questions have 4 answer choices (a, b, c, or d). How many ways can you answer the test if you leave an answer for each question?

7) Telephone numbers is the United States begin with three-digit area codes followed by seven-digit local telephone numbers. Area codes and local telephone numbers cannot begin with a 0 or 1. How many telephone numbers are possible?

8) You have been put in charge of a concert tour. On one of the days, you have six bands that will all share the stage one after another. How many ways can you put together a
6-band concert?
3-band concert?

9) Five men and five women are to line up at a checkout stand. In how many ways can they line up if the line must start with a woman and the people in line alternate according to the sequence WMWMWM....?