Section 9.1

Problem Solving with algebra

Evaluate each of the following algebraic expressions for x = 14 and n = 28

1. $15 + 3x$
2. $4n - 6$
3. $\frac{n}{7} + 20$
4. $6x \div 12$

Now try this one… 5. $n \div 2x$

Properties of equality
Addition and subtraction: add or subtract the same number from both sides of an equation
Multiplication and division: multiply or divide each term of an equation by the same non-zero number
Simplification: replacing an expression in an equation by an equivalent expression

Solve each equation using the properties of equality and simplification

1. $5x - 9 = 2x + 15$
2. $x + \frac{1}{7}x = 19$
3. $2(x - 5) = -3(x + 1)$

Solving inequalities using the properties and simplification.
Inequalities are solved the same way as equations with one distinct difference—If you multiply or divide by a negative number when using the properties, you must reverse the inequality sign.

Solve and *graph* each inequality

1. $4(3x) + 16 < 52$
2. $11x - 7 \leq 3x + 23$

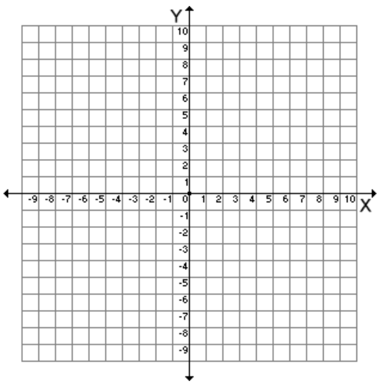
Using Algebra as a Problem Solving Strategy

The manager of a garden center wants to order a total of 138 trees consisting of two types: Japanese red maple and flowering pears. Each maple tree costs \$156 and each pear tree costs \$114. If the manager has a budget of \$18,000, which must all be spent for the trees, how many maple trees will he order?

Section 9.2

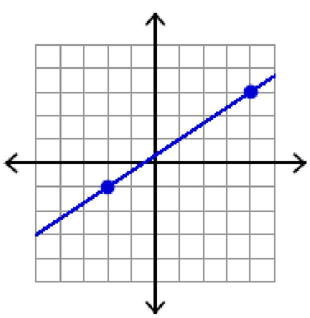
Coordinates, Slope & Lines

Rectangular Coordinates and the Coordinate plane



The slope of a line or line segment is defined as the steepness of the line. It can be found using the equation $\frac{rise}{run}$. Start at any point on the line and travel straight up or down, then go right or left to connect it to the other point. Find the slope of the following lines…

1.



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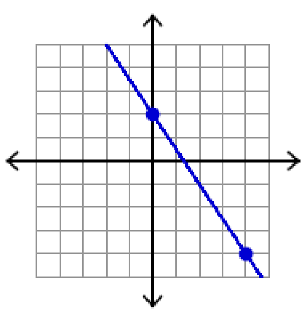
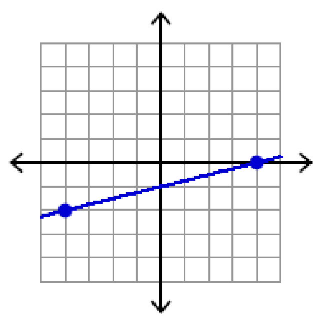


3.



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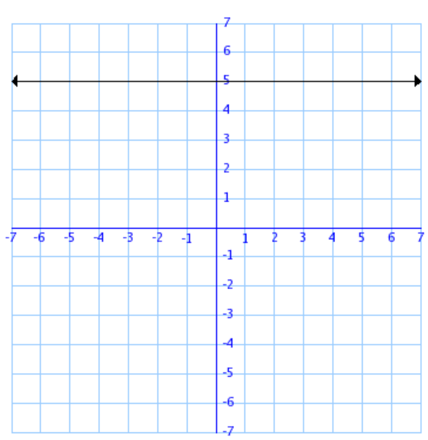
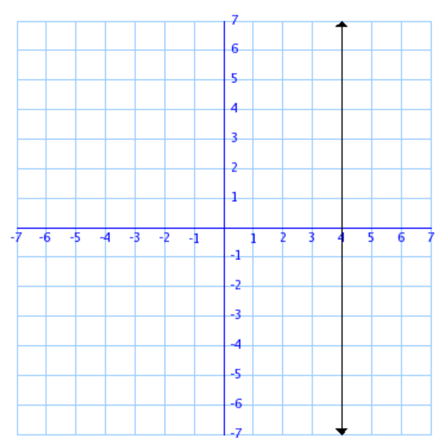
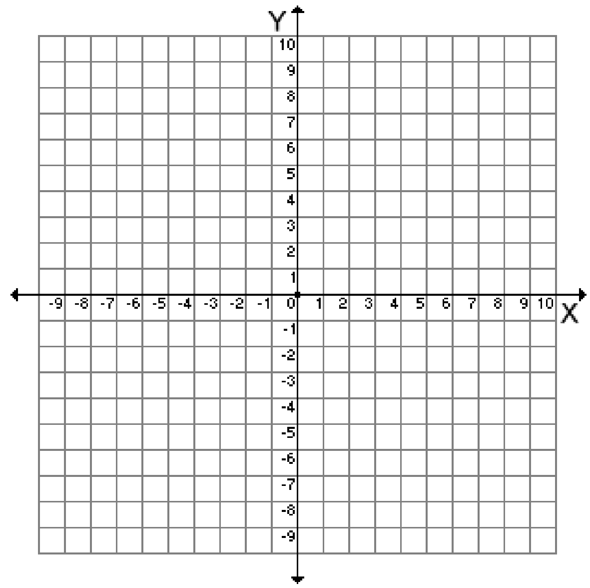


5.



You can also find the slope of a line by using two points on the line without a graph.

6. Find the slope of a line going through the points (2,9) and (5,3) then graph the line on the plane below.

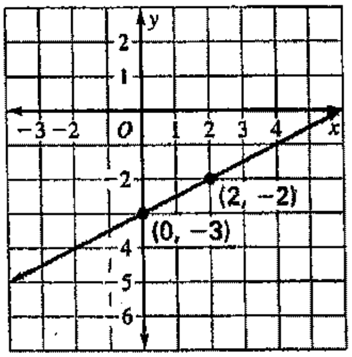


Writing equations in slope-intercept form.

$$y = mx + b$$

1) Write an equation of the line with a slope of –13 and a y–intercept of 19.

2) Write an equation of the line shown.



3) Write an equation of a line that passes through the points (0,7) and (4, –8)

4) Write an equation of a line that passes through the points (–2,9) and (3, –5)

5) Write an equation of a line that is parallel to the line y = –15x + 11 and passes through the point (–1,4).

6) Write an equation of a line that is perpendicular to the line y = –9x + 6 and passes through the point (6, –4).

Solving systems of equations using substitution

$y = x - 6$
$x + y = -2$

$2x + 5y = -4$
$3x - 2y = 13$

To put in a garden, you rent a rototiller. There are two local equipment rental agencies and they charge flat fees and then by the half-day. The breakdown for each rental center is below.

Dig It: $20 stock fee
$15 for half-day

Engines-R-Us: $20 stock fee
$17.50 for half-day

Which company offers the best deal and for how many half-days is this the best deal?

Section 9.3

Functions & Graphs

Section 2.2 Functions, Coordinates and Graphs

Find a rule for each table

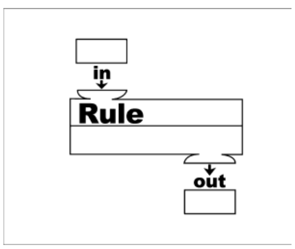
x	1	2	8	5	0
y	8	13	43	28	3

x	5	1	6	2	9
y	26	2	37	5	82

A **function** is 2 sets and a rule that assigns each element of the first set to exactly one element of the second set.

The two sets of the functions have names. The first set is called the domain, and the second set is called the range.

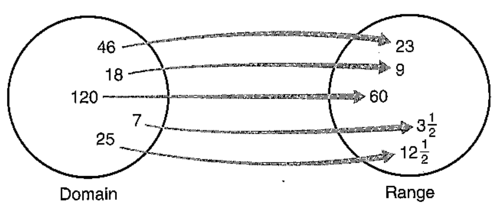
Usually in elementary grades, children see pictures of a function machine and see the in and out.



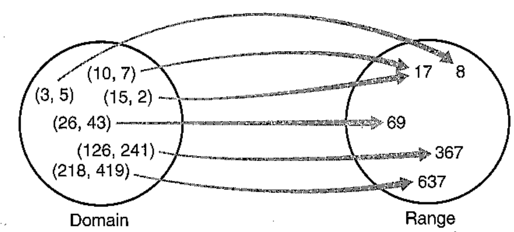
Another way relationships with functions are shown is by using arrow diagrams.

Describe a rule for assigning each element of the domain to an element of the range for the following arrow diagrams.

1.



2.



Sometimes we are asked to tell whether the elements of the domain have a one-to-one correspondence with the elements of the range. Here we are being asked whether or not the two sets determine rules which are functions. Determine if each of the following rules for the given sets are functions. If it is not a function, explain why.

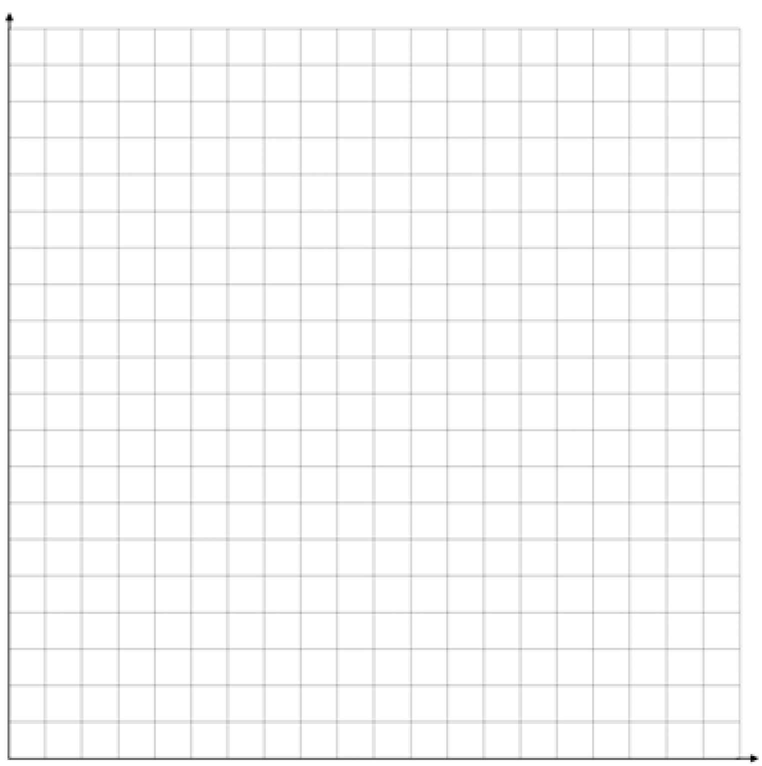
1) Each person is assigned to his or her social security number.
2) Each amount of money is assigned to the object it will buy.
3) Each person is assigned to a person who is older.
4) Each pencil is assigned to its length.

Write an algebraic rule for each of the following functions, where the domain is all whole numbers and x represents an element in the domain.

1) *f(x)* is an element in the range, and each element in the domain is assigned to 3 more than twice its value.
2) *g(x)* is an element in the range, and each element in the domain is assigned to 1 more than four times its value.
3) *h(x)* is an element in the range, and each element in the domain is assigned to 10 times its value.
4) Evaluate *f(45), g(56) & h(84)*

A taxi meter starts at $1.60 and increases at a rate of $1.20 for every minute. Let x represent the number of minutes and f(x) represent the total cost.

a) Write an equation for the total cost of a taxi ride for x number of minutes.
b) What is the value of f(15)?
c) Graph this function.

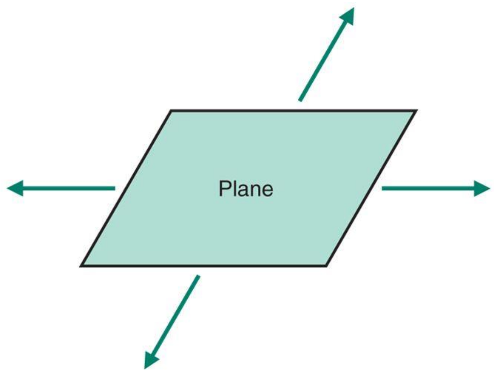


Section 10.1 Plane Figures

POINTS, LINES, AND PLANES

A **line** is a set of points that we describe intuitively as being "straight" and extending indefinitely in both directions. The line in below passes through points *A* and *B* and is denoted by $\overleftrightarrow{AB}$. The arrows indicate that the line continues indefinitely in both directions. If two or more points are on the same line, they are called **collinear.**

A plane is another set of points that is undefined. We describe a **plane** as being "flat" like the top of a table, but extending indefinitely. The surfaces of floors and walls are other common models for portions of planes. A plane can be illustrated by a drawing that uses arrows, as in the figure below.



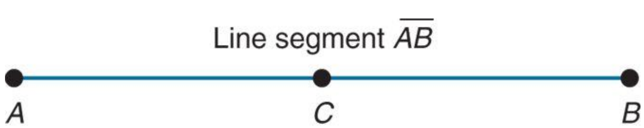
A standard sheet of paper is a model for part of a plane.

1. What part of a sheet of paper might be used as a model for a line?
2. What part of a sheet of paper might be used as a model for a point?
3. How can models of lines and points be obtained by folding a sheet of paper?

Points, lines, and *planes* are undefined terms in geometry that are used to define other terms and geometric figures. The following paragraphs contain some of the more common definitions and examples of figures that occur in planes.

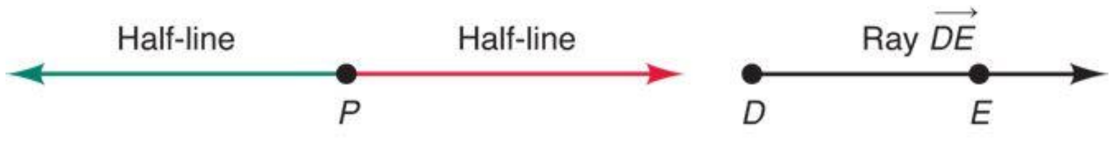
Line Segments

A **line segment** consists of two points on a line and all the points between them. The line segment with **endpoints** *A* and *B* is denoted by $\overline{AB}$. It is customary to refer to the length of a line segment by removing the bar above the letters. So if segment $\overline{AB}$ is equal in length to segment $\overline{BC}$, we write $AB = BC$. To **bisect** a line segment means to divide it into two parts of equal length. The **midpoint** *C* bisects $\overline{AB}$.



Half-Lines and Rays

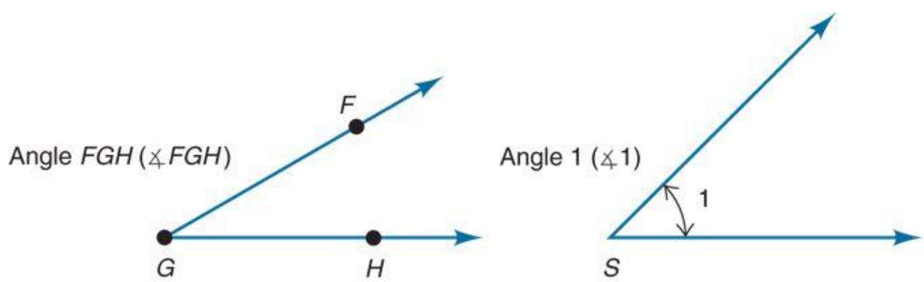
A point on a line partitions the line into three disjoint sets: the point and two **half-lines.** The figure below shows two half-lines that are determined by point *P*. A **ray** consists of a point on a line and all the points in one of the half-lines determined by the point. The ray in part b, which has *D* as an *endpoint* and contains point *E*, is denoted by $\overrightarrow{DE}$.



(a) (b)

Angles

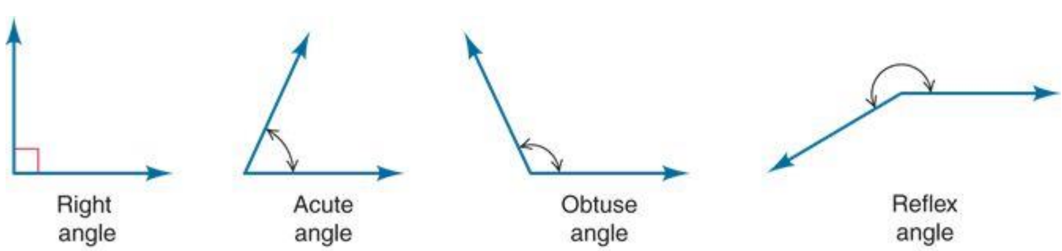
An **angle** is formed by the union of two rays, with a common endpoint, as shown in the figures below, or by two line segments that have a common endpoint, as in part b. This endpoint is called the **vertex,** and the rays or line segments are called the **sides of the angle.** The angle with vertex *G*, whose sides contain points *F* and *H*, is denoted by ________.
Sometimes it is convenient to identify an angle by the letter of its vertex, such as _________in part a, or by a numeral, such as _________ in part b. If two angles have the same measure, we write, for example, _________ = __________.



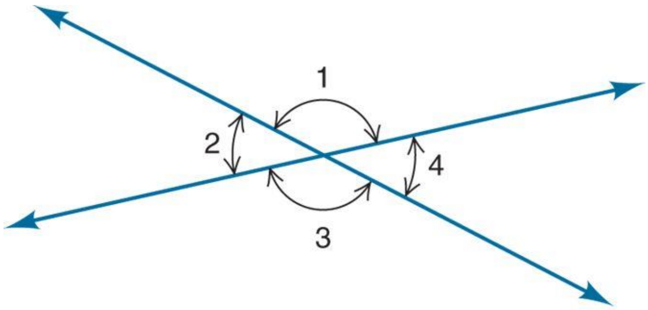
(a) (b)

ANGLE MEASUREMENTS

If an angle has a measure of 90°, it is called a ________________**;** if it is less than 90°, and greater than 0°, it is called an _________________**;** if it is greater than 90° and less than 180°, it is called an ________________**;** and if it has a measure of 180° it is called a _________________**.** It is customary to draw a ___________ at the vertex of a right angle. Occasionally we use angles with measures of more than 180° and less than 360°, such an angle is called a ____________________**.** To indicate a reflex angle, we draw a circular arc to connect the two sides of the angle.



(a) (b) (c) (d)



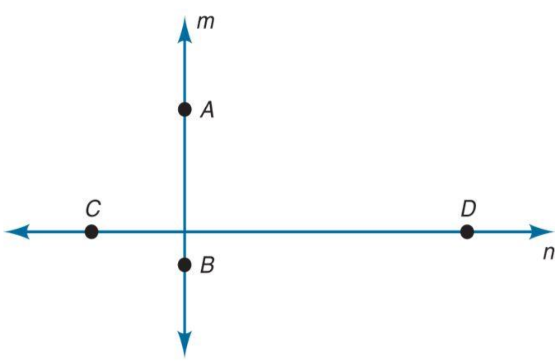
Vertical angles
$\measuredangle 2$ and $\measuredangle 4$; $\measuredangle 1$ and $\measuredangle 3$

Two intersecting lines form four pairs of adjacent supplementary angles. For example, $\measuredangle 1$ and $\measuredangle 4$ in the figure below are supplementary angles. Nonadjacent angles formed by two intersecting lines, such as $\measuredangle 2$ and $\measuredangle 4$ are called **vertical angles** and vertical angles have equal measure.

Supplementary angles are pairs of angles that have a sum of 180 degrees. Name 3 pairs of supplementary angles from the diagram above.

________________ ________________ ________________

PERPENDICULAR AND PARALLEL LINES



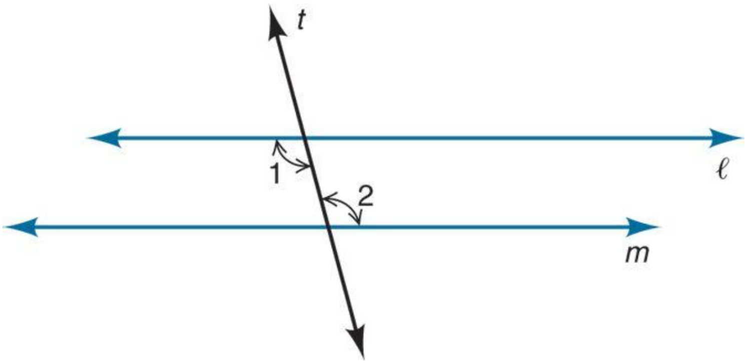
Perpendicular lines

If two lines intersect to form right angles, they are **perpendicular.** Lines *m* and *n* in the figure to the left are perpendicular; this is indicated by writing $m \perp n$. Two intersecting line segments, such as $\overline{AB}$ and $\overline{CD}$ to the left are **perpendicular line segments** if they lie on perpendicular lines. In this case, we write $\overline{AB} \perp \overline{CD}$.

If two lines are in a plane and they do not intersect, they are **parallel.** Lines *m* and *n* below are parallel; this is indicated by writing $m \parallel n$. Similarly, two line segments are **parallel line segments** if they lie on parallel lines. For example, segments $\overline{EF}$ and $\overline{GH}$ are parallel, and we write $\overline{EF} \parallel \overline{GH}$.

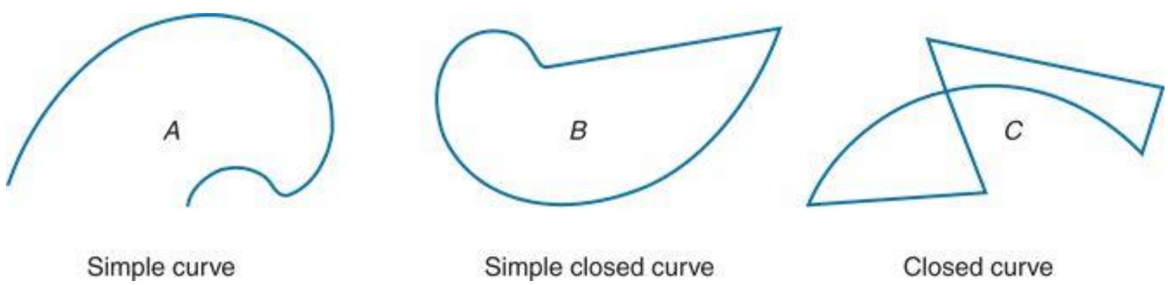
Parallel lines

If two lines ℓ and *m* are intersected by a third line *t* , we call line *t* a ____________________. Two very special angles are created on the alternate sides of the transversal and are interior to lines ℓ and *m*. These angles are called __________________________**.** If the two lines ℓ and *m* are parallel, *the alternate interior angles have the same measure.* The converse of this statement is also true: *If the alternate interior angles have the same measure,* lines ℓ and *m* are parallel.

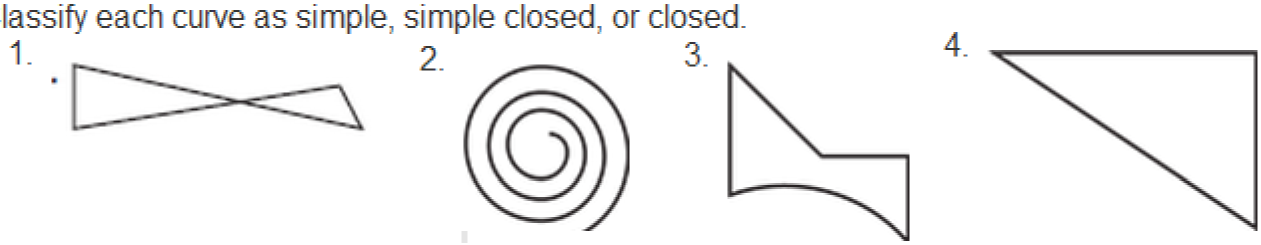


CURVES AND CONVEX SETS

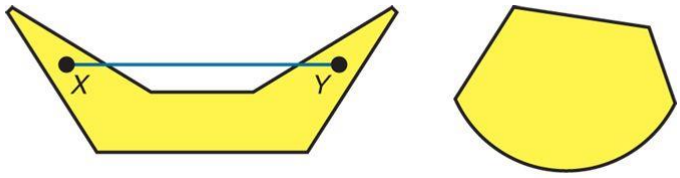
Several types of curves are shown below. Curve *A* is called a **simple curve** because it starts and stops without intersecting itself. Curve *B* is a **simple closed curve** because it is a simple curve that starts and stops at the same point. Curve *C* is a **closed curve,** but since it intersects itself, it is not a simple closed curve.



Classify each curve as simple, simple closed, or closed.



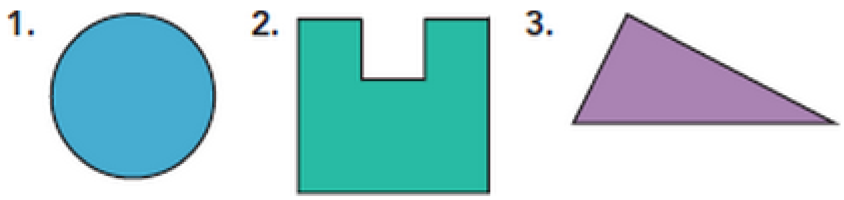
Convex Sets The union of a simple closed curve and its interior is called a **plane region.** Plane regions can be classified as *concave* or *convex*. We say that a set is **concave** if it contains two points such that the line segment joining the points does not completely lie in the set. If a set is not concave, it is called **convex.**



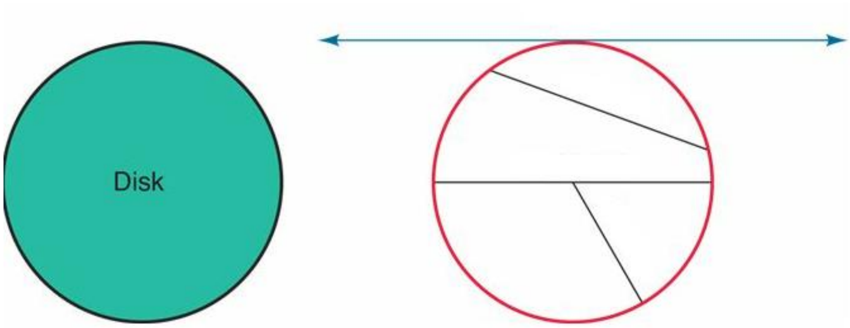
Concave
(a)

Convex
(b)

Classify each region as concave or convex

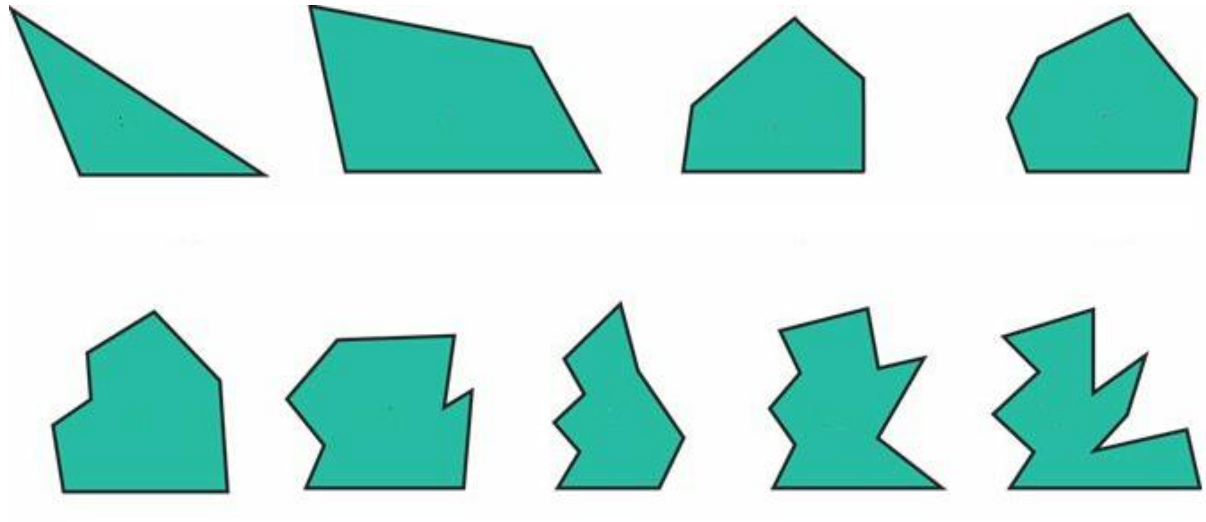


Circles A **circle** is a special case of a simple closed curve whose interior is a convex set. Each point on a circle is the same distance from a fixed point, called the **center.** A line segment from a point on the circle to its center is a **radius,** and a line segment whose endpoints are both on the circle is a **chord.** A chord that passes through the center is a **diameter.** The words *radius* and *diameter* are also used to refer to the lengths of these line segments. A line that intersects the circle in exactly one point is a **tangent.** The distance around the circle is the **circumference.** The union of a circle and its interior is called a **disk.**

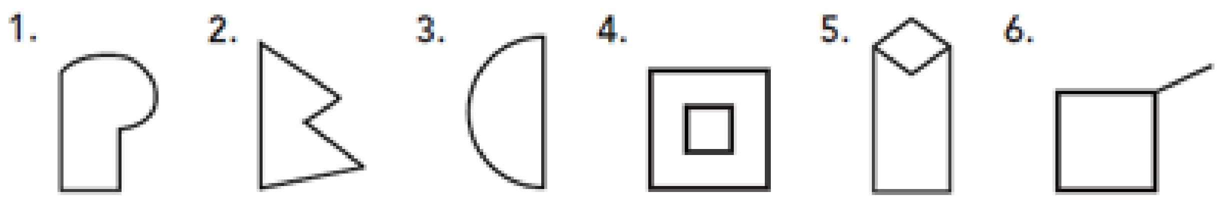


POLYGONS

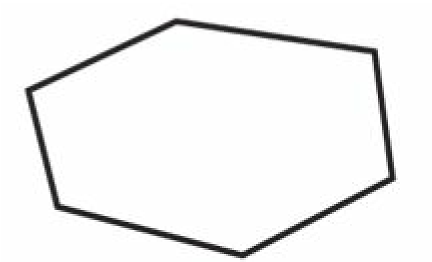
A **polygon** is a simple closed curve that is the union of line segments. The union of a polygon and its interior is called a **polygonal region.** Polygons are classified according to their number of line segments. The line segments of a polygon are called **sides,** and the endpoints of these segments are **vertices.** Two sides of a polygon are **adjacent sides** if they share a common vertex, and two vertices are **adjacent vertices** if they share a common side.



Which of the following figures are polygons?



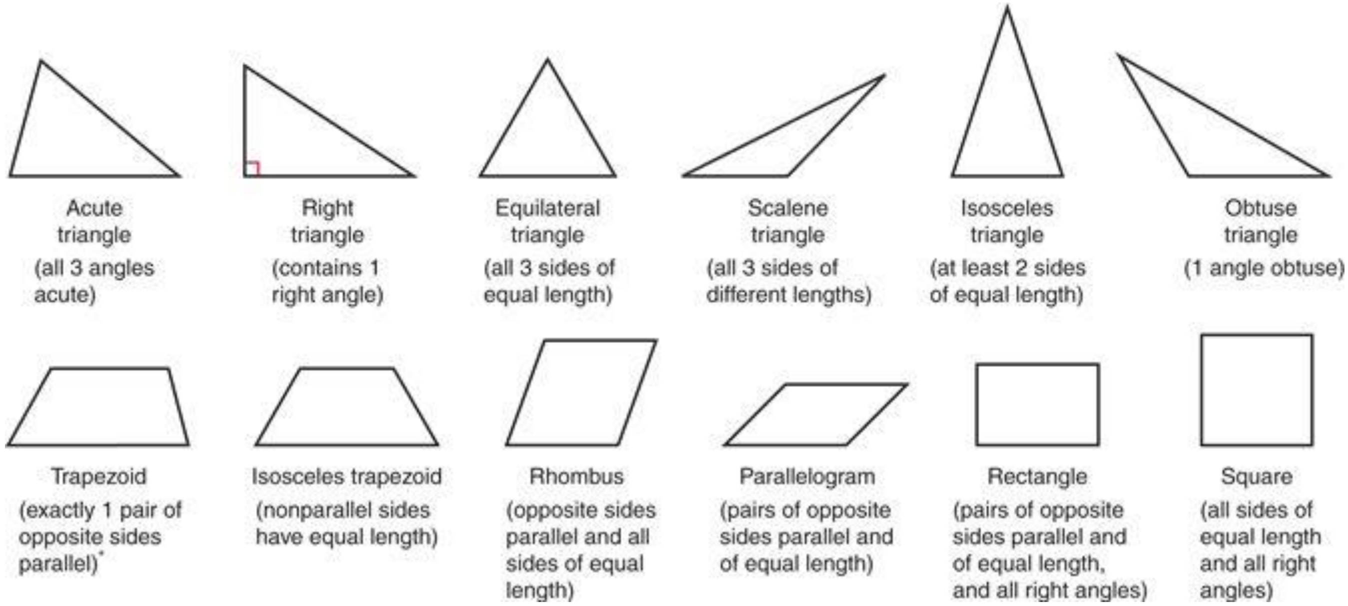
Any line segment connecting one vertex of a polygon to a nonadjacent vertex is a _______________.



How many diagonals are there in each of the following polygons?

1. Quadrilateral
2. Triangle
3. Pentagon

Certain triangles and quadrilaterals occur often enough to be given special names.



Determine whether each statement is true or false, and state a reason.

1. Every square is a rectangle.
2. Every equilateral triangle is an isosceles triangle.
3. Some right triangles are isosceles triangles.
4. Some trapezoids are parallelograms.
5. Some isosceles triangles are scalene triangles.

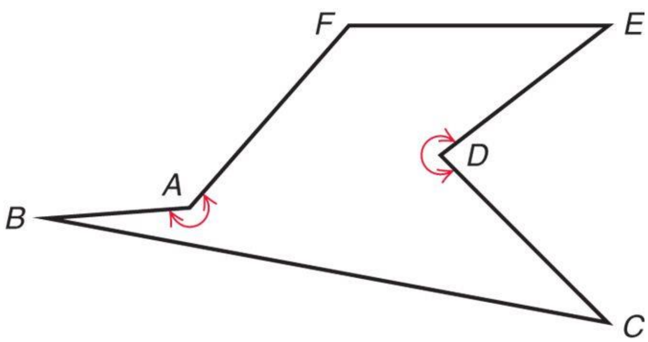
Problem

How many diagonals does a 15-sided polygon have?

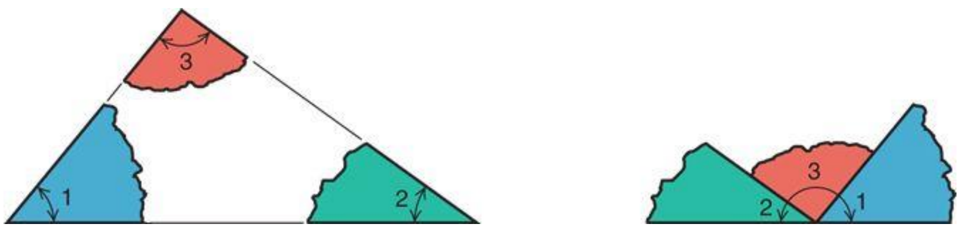
Section 10.2 Polygons and Tessellations

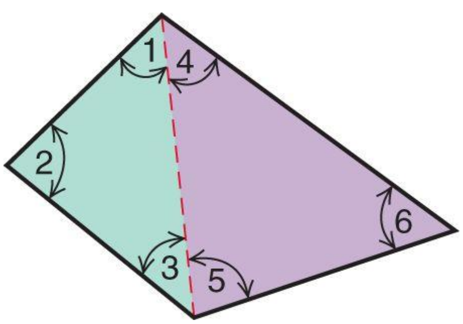
ANGLES IN POLYGONS

The vertex angles of a polygon with four or more sides can be any size between 0° and 360°. In the hexagon to the right, ∡*B* is less than 20° and ∡ *D* and ∡*A*are both greater than 180°. In spite of this range of possible sizes, there is a relationship between the sum of all the angles in a polygon and its number of sides.



In any triangle, the sum of the three angle measures is 180°. This fact was proved by Greek mathematicians in the fourth century B.C.E. One way of demonstrating this theorem is to draw an arbitrary triangle and cut off its angles, as shown below. When these angles are placed side by side with their vertices at a point, they form one-half of a revolution (180°) about the point.

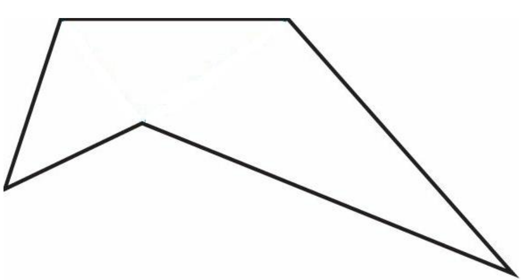


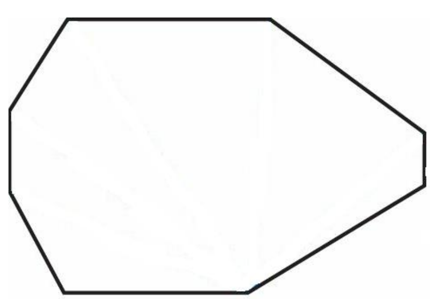


The sum of the angles in a polygon of four or more sides can be found by subdividing the polygon into triangles so that the vertices of the triangles are the vertices of the polygon.

Find the sum of all the angles in each polygon by dividing the polygon into triangles.

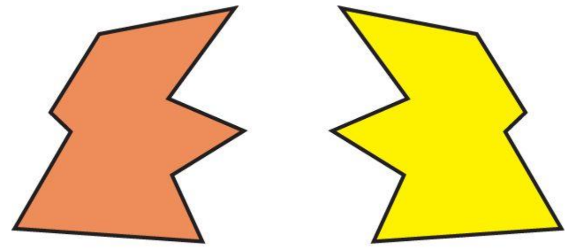
1. Pentagon
2. Octagon





CONGRUENCE

The idea of congruence is quite simple to understand intuitively: Two plane figures, such as those below, are **congruent plane figures** if one can be placed on the other so that they coincide. Another way to describe congruent plane figures is to say that they have the same size and shape.



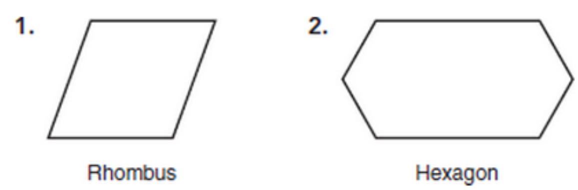
Congruent plane figures

REGULAR POLYGONS

Sections 9.1 and 9.2 opened with photographs of hexagons and triangles that grow naturally with congruent line segments and congruent angles. The figures in those photographs are examples of regular polygons. A polygon is called a **regular polygon** if it satisfies both of the following conditions:

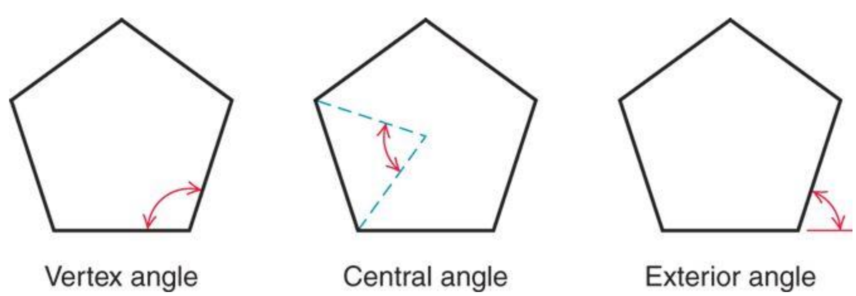
1. All _______________ are congruent.
2. All ____________________ are congruent.

The figures to the right satisfy only one of the two conditions for regular polygons. For each polygon determine which condition is satisfied and which condition is not satisfied.



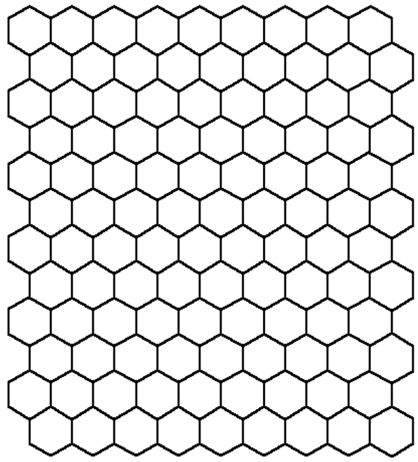
FINDING ANGLES IN REGULAR POLYGONS

There are three special angles in regular polygons. A **vertex angle** is formed by two adjacent sides of the polygon; a **central angle** is formed by connecting the center of the polygon to two adjacent vertices of the polygon; and an **exterior angle** is formed by one side of the polygon and the extension of an adjacent side.



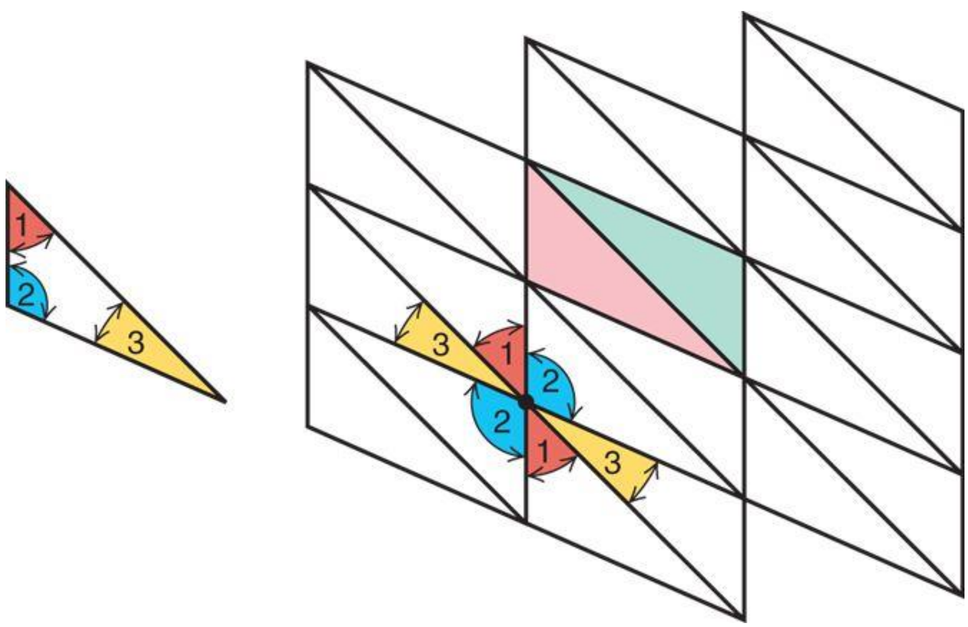
TESSELLATIONS WITH POLYGONS

The hexagonal cells of a honeycomb provide another example of regular polygons in nature. The cells in this photograph show that regular hexagons can be placed side by side with no uncovered gaps between them. Any arrangement in which non overlapping figures are placed together to entirely cover a region is called a **tessellation.** Floors and ceilings are often *tessellated,* or *tiled,* with square-shaped material, because squares can be joined together without gaps or overlaps. Equilateral triangles are also commonly used for tessellations. These three types of polygons—*regular hexagons, squares, and equilateral triangles—are the only regular polygons that will tessellate.*

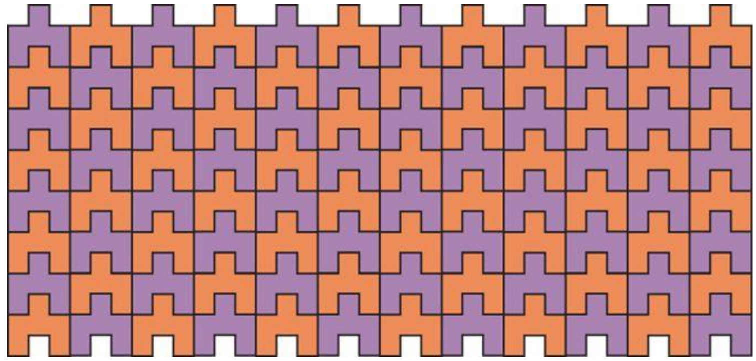


Ask about Subway cheese. http://youtu.be/uH9pnTg4_O8

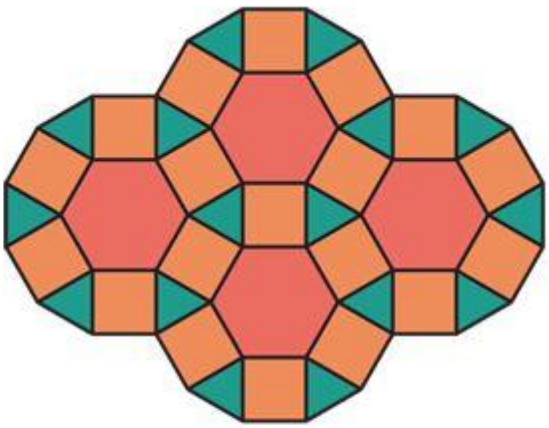
The triangle is an easy case to consider first. You can see that any triangle will tessellate by simply putting two copies of the triangle together to form a parallelogram. Copies of the parallelogram can then be moved horizontally and vertically. The points at which the vertices of the triangle meet are called the **vertex points** of the tessellation. Since the sum of the angles in a triangle is 180°, the 360° about each vertex point of the tessellation will be covered by using each angle of the triangle twice.



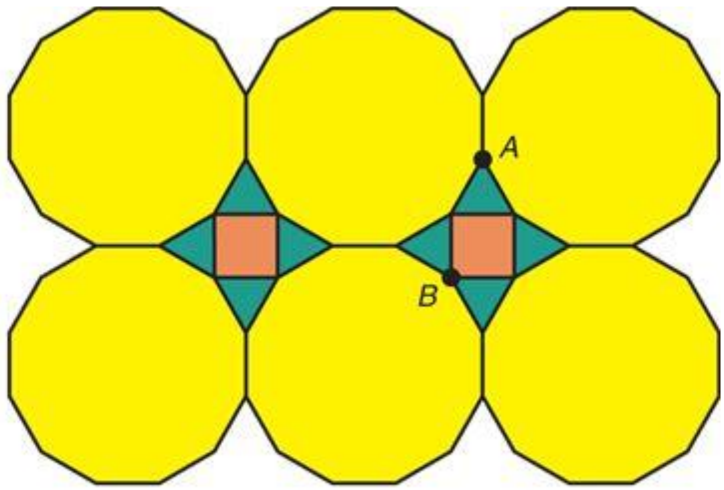
The quadrilateral in the tessellation to the right is concave. It is quite surprising that every quadrilateral, convex or concave, will tessellate. This is not true for polygons with more than four sides. Although there are some pentagons that will tessellate, there are others that will not tessellate. Similarly, some hexagons will tessellate (for example, a regular hexagon), but not all hexagons will.



The only regular polygons that will tessellate by themselves are the equilateral triangle, the square, and the regular hexagon. However, if we allow two or more regular polygons in a tessellation, there are other possibilities. Two such tessellations are shown below. The tessellation in part a uses three different regular polygons.



(a)



(b)

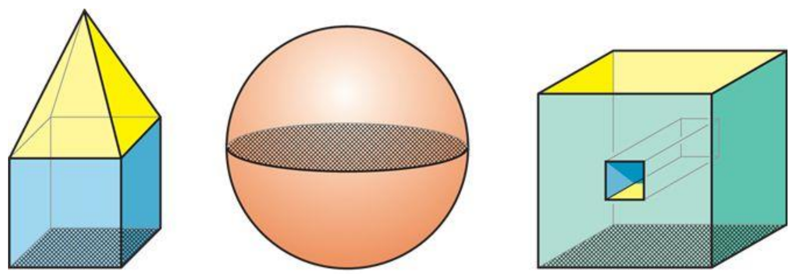
Finding the sum of the interior vertex angles of a polygon.

Find the sum of the interior vertex angles if a 50-sided polygon

Section 10.3 Space Figures

POLYHEDRA

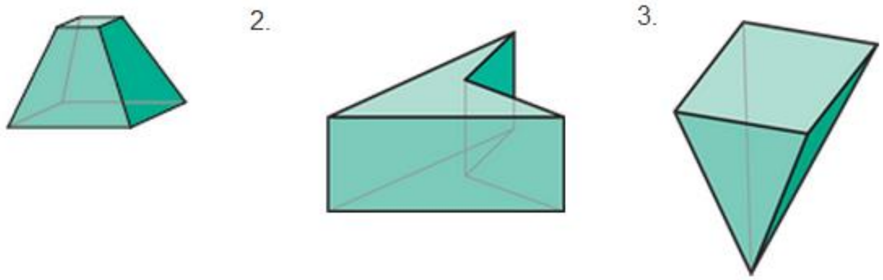
The surface of a figure in space whose sides are polygonal regions, such as the one below, is called a **polyhedron** (*polyhedra* is the plural). The polygonal regions are called **faces,** and they intersect in the **edges** and **vertices** of the polyhedron. The union of a polyhedron and its interior is called a **solid.** The figures below show examples of a polyhedron and two figures that are not polyhedra. The figure in part a is a polyhedron because its faces are polygonal regions. The figures in parts b and c are not polyhedra because one has a curved surface and the other has two faces that are not polygons.



(a) (b) (c)

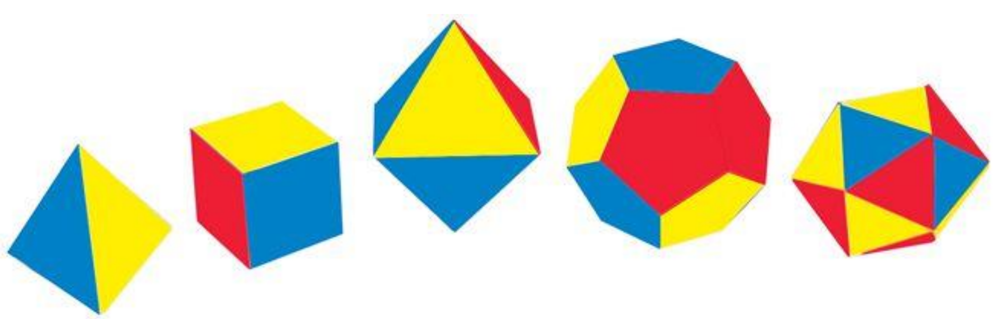
A polyhedron is **convex** if the line segment connecting any two of its points is contained inside the polyhedron or on its surface. Classify the following polyhedra as convex or concave.

1. 2. 3.



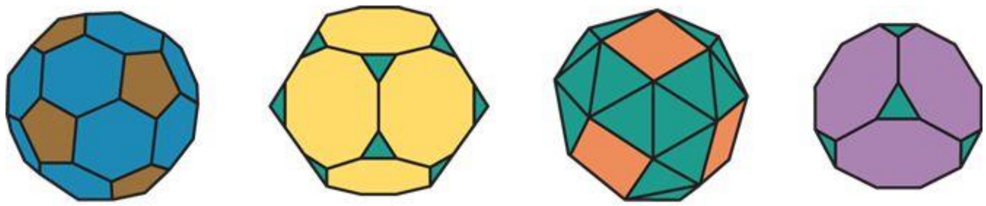
REGULAR POLYHEDRA

The best known of all the polyhedra are the *regular polyhedra,* or *Platonic solids.* A **regular polyhedron** is a convex polyhedron whose faces are *congruent regular polygons,* the same number of which meet at each vertex. The ancient Greeks proved that there are only five regular polyhedra. Models of these polyhedra are shown below.
The **tetrahedron** has 4 triangles for faces; the **cube** has 6 square faces; the **octahedron** has 8 triangular faces; the **dodecahedron** has 12 pentagons for faces; and the **icosahedron** has 20 triangular faces. **POLY EXE**



Semiregular Polyhedra Some polyhedra have two or more different types of regular polygons for faces.

Several other semiregular polyhedra are shown below. You may recognize the combination of hexagons and pentagons in part a as the pattern used on the surface of soccer balls.



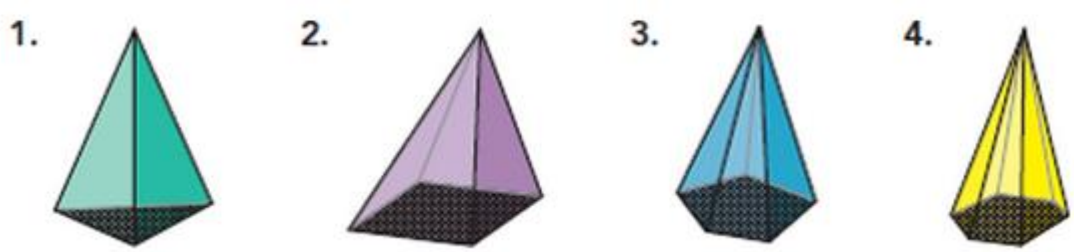
(a) (b) (c) (d)

PYRAMIDS AND PRISMS

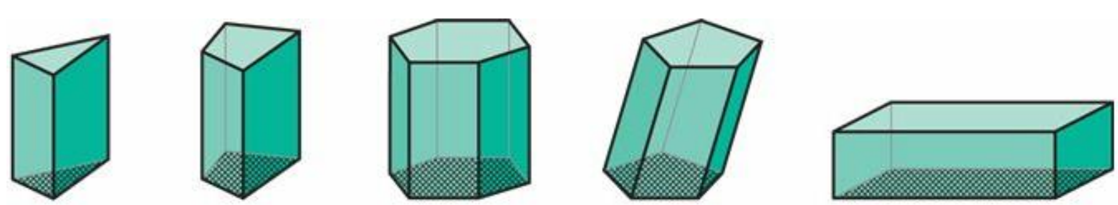
Chances are that when you hear the word *pyramid,* you think of the monuments built by the ancient Egyptians. Each of the Egyptian pyramids has a square base and triangular sides rising up to the vertex. This is just one type of pyramid. In general, the **base of a pyramid** can be any polygon, but its sides are always triangular. Pyramids are named according to the shape of their bases. Church spires are familiar examples of pyramids. They are usually square, hexagonal, or octagonal pyramids.

The vertex that is not contained in the pyramid's base is called the **Apex**. A pyramid that does not have its apex directly above the center of its base is said to be **Oblique** pyramid.

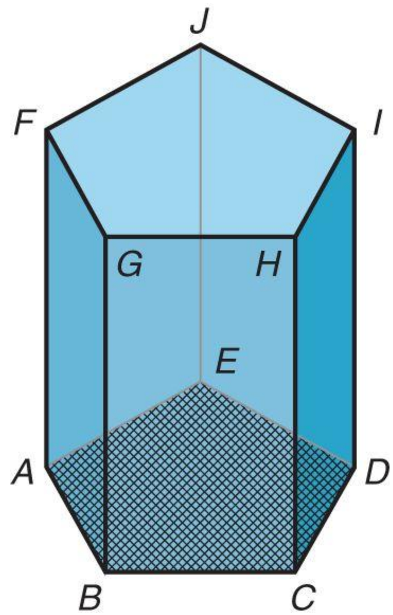
Classify each pyramid below.



Prisms Prisms are another common type of polyhedron. A **prism** has two parallel **bases,** upper and lower, which are congruent polygons. Like pyramids, prisms get their names from the shape of their bases. If the lateral sides of a prism are perpendicular to the bases, it is called a **right prism.** A rectangular prism, which is modeled by a box, is the most common type of prism. If some of the lateral faces of a prism are parallelograms that are not rectangles, as in the pentagonal prism, the prism is called an **oblique prism.** The union of a prism and its interior is called a **solid prism.** A rectangular prism that is a solid is sometimes called a **rectangular solid.** *Name each prism below.*



The figure to the right is a right prism with bases that are regular pentagons.



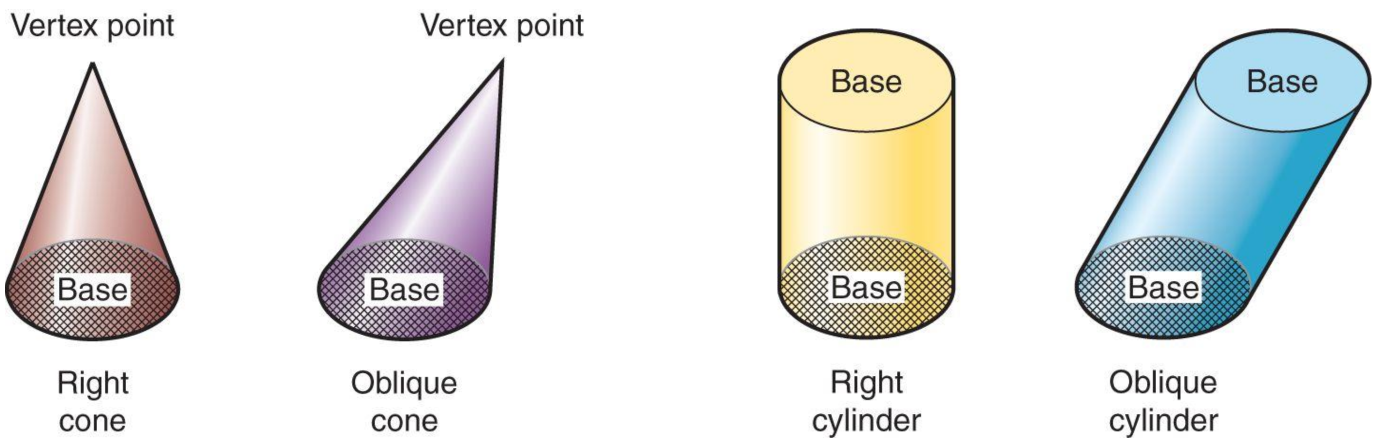
1. What is the measure of the dihedral angle between face *ABGF* and face *BCHG*?

2. What is the measure of the dihedral angle between face *GHIJF* and face *CDIH*?

3. Name two faces that are in parallel planes

CONES AND CYLINDERS

Cones and cylinders are the circular counterparts of pyramids and prisms. Ice cream cones, paper cups, and party hats are common examples of cones. A cone has a circular region (disk) for a **base** and a lateral surface that slopes to the **vertex (apex).** If the vertex lies directly above the center of the base, the cone is called a **right cone** or usually just a cone; otherwise, it is an **oblique cone**.



Ordinary cans are models of cylinders. A **cylinder** has two parallel circular **bases** (disks) of the same size and a lateral surface that rises from one base to the other. If the centers of the upper base and lower base lie on a line that is perpendicular to each base, the cylinder is called a **right cylinder** or simply a cylinder; otherwise, it is an **oblique cylinder**. Almost without exception, the cones and cylinders we use are right cones and right cylinders.

PROBLEM-SOLVING APPLICATION

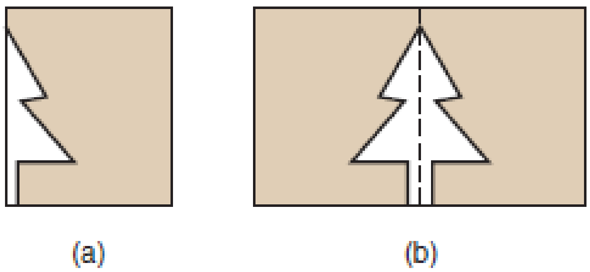
There is a remarkable formula that relates the numbers of vertices, edges, and faces of a polyhedron. This formula was first stated by René Descartes about 1635. In 1752 it was discovered again by Leonhard Euler and is now referred to as **Euler's formula.** See if you can discover this formula, either before or as you read the parts of the solution presented below. Use Polyexe again

What is the relationship among the numbers of faces, vertices, and edges of a polyhedron?

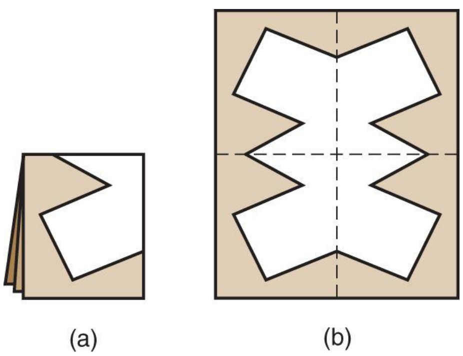
Section 10.4 Symmetric Figures

REFLECTION SYMMETRY FOR PLANE FIGURES

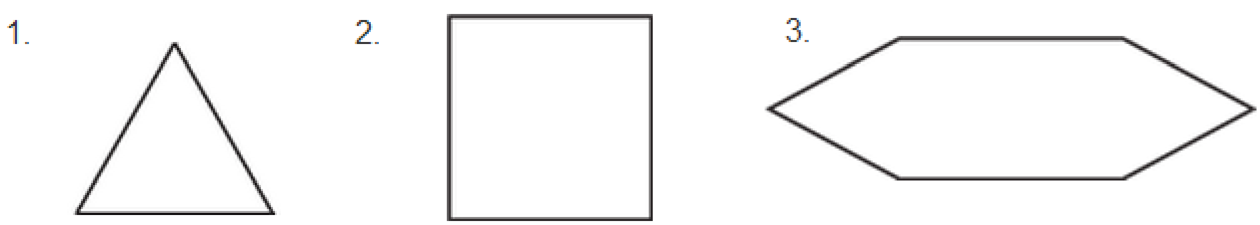
Many years before it became popular to teach geometric ideas in elementary school, cutting out symmetric figures was a common classroom activity. The procedure is to fold a piece of paper and draw a figure that encloses part of, or all of the crease, as shown below. When the figure is cut out and unfolded, it is symmetric (see part b). The crease is called a **line of symmetry,** and the figure is said to have **reflection symmetry.**



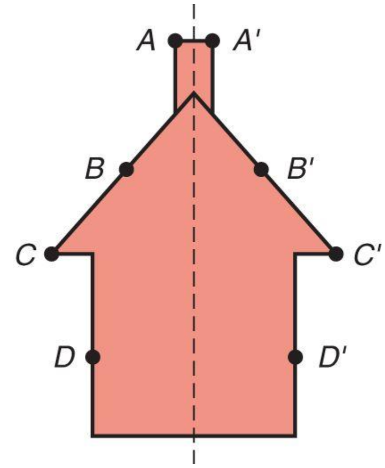
Some figures have more than one line of symmetry. To produce a figure with two such lines, fold a sheet of paper in half and then in half again. Then draw a figure whose endpoints touch the creases. If the figure is cut out and the paper is opened, the two perpendicular creases will be lines of symmetry for the figure, as shown below.



Each of the following polygons has two or more lines of symmetry. Determine these lines for each figure.

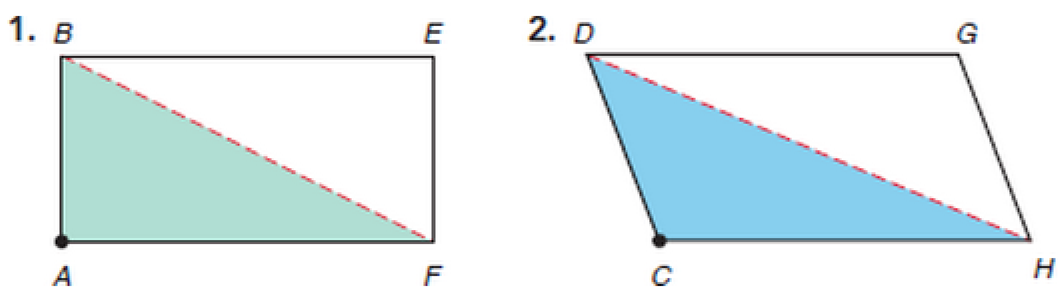


The idea of symmetry can be made more precise by adopting the term *image,* which is suggested by mirrors. If a line can be drawn through a figure so that each point on one side of the line has a matching point on the other side at the same perpendicular distance from the line, it is a **line of symmetry.** If two points on opposite sides of this line match up, one is called the **image** of the other. A few points and their images have been labeled on the figure to the right, where *A* corresponds to *A′*, *B* to *B′*, *C* to *C′*, and *D* to *D′*. Each line segment connecting a point and its image is perpendicular to the line of symmetry.



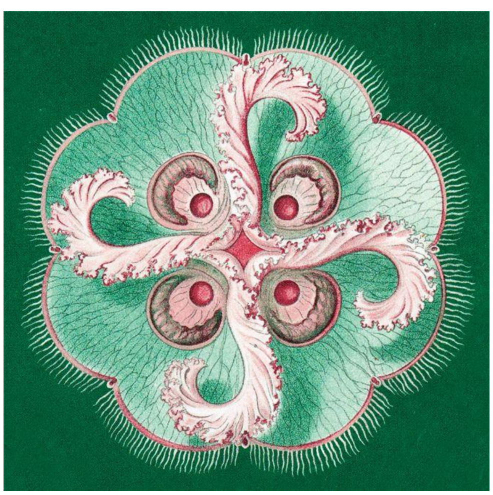
False reflections

For each of the following figures, show that the diagonal (red line) is not a line of symmetry by finding the image of points *A* and *C* for reflections about the diagonals.

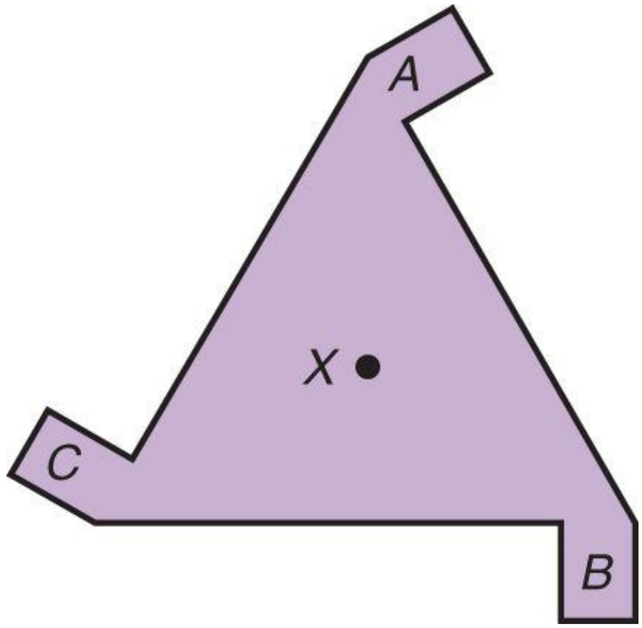


ROTATION SYMMETRY FOR PLANE FIGURES

The figure below may look like a drawing of a plant, but it is a drawing of a type of jellyfish called *Aurelia.* It seems to have the form and balance of a symmetric figure, but it has no lines of reflection. It does, however, have **rotation symmetry,** because it can be turned about its center so that it coincides with itself. For example, if it is rotated 90° clockwise, the top “arm” will move to the 3 o'clock position, the bottom “arm” will move to the 9 o'clock position, etc.



Let's consider another example of rotation symmetry. If the figure below is cut out and held down by a pencil at point *X*, the top figure can be rotated clockwise so that *A* goes to *B*, *B* to *C*, and *C* to *A*. This is an example of rotation symmetry, and *X* is called the **center of rotation.** Since the figure is rotated 120° (one-third of a full turn), it has a *120° rotation symmetry*. From its original position, this figure can also be made to coincide with itself after a 240° clockwise rotation, with *A* going to *C*, *B* to *A*, and *C* to *B*. This is a *240° rotation symmetry*. Since the figure can be rotated back onto itself after a 360° rotation, the figure also has a *360° rotation symmetry. Note:*Any figure can be rotated 360° by using any point as the center of rotation. Thus, we will be interested in a 360° rotation symmetry only when a figure has other rotation symmetries as well.



Some figures have both reflection symmetry and rotation symmetry. The regular polygons have both types. The central angles of these polygons determine the angles for the rotation symmetries.

Find all the reflection and rotation symmetries for a regular hexagon.

PROBLEM-SOLVING APPLICATION

For every plane figure with two or more reflection symmetries, there is a relationship between the number of these symmetries and the number of rotation symmetries. What is this relationship? Use the shapes below to get you started.